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On Achievable Rates and Complexity of LDPC Codes for Parallel Channels with Application to Puncturing

Igal Sason Gil Wiechman

Technion – Israel Institute of Technology
Haifa 32000, Israel
{sason@ee, igillw@tx}.technion.ac.il

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Abstract

This paper considers the achievable rates and decoding complexity of low-density parity-check (LDPC) codes over statistically independent parallel channels. The paper starts with the derivation of bounds on the conditional entropy of the transmitted codeword given the received sequence at the output of the parallel channels; the component channels are considered to be memoryless, binary-input, and output-symmetric (MBIOS). These results serve for the derivation of an upper bound on the achievable rates of ensembles of LDPC codes under optimal maximum-likelihood (ML) decoding when their transmission takes place over parallel MBIOS channels. The paper relies on the latter bound for obtaining upper bounds on the achievable rates of ensembles of randomly and intentionally punctured LDPC codes over MBIOS channels. For ensembles of punctured LDPC codes, the calculation of bounds on their thresholds under ML decoding and their exact thresholds under iterative decoding (based on density evolution analysis) is of interest in the sense that it enables to assess the degradation in the asymptotic performance which is attributed to the sub-optimality of iterative decoding (as compared to ML decoding), and also to assess the inherent loss in the asymptotic performance which is attributed to the structure of these ensembles, even if ML decoding could be applied to decode LDPC codes. The paper also provides a lower bound on the decoding complexity (per iteration) of ensembles of LDPC codes under message-passing iterative decoding over parallel MBIOS channels; the bound is given in terms of the gap between the rate of these codes for which reliable communication is achievable and the channel capacity. Similarly to the case of a single MBIOS channel, this lower bound grows like the log of the inverse of the gap to capacity. The latter bound is used for the derivation of lower bounds on the decoding complexity of punctured LDPC codes over MBIOS channels; looser versions of these bounds suggest a simplified re-derivation of previously reported bounds on the decoding complexity of randomly punctured LDPC codes. The paper presents a diagram which shows interconnections between the theorems introduced in this paper and some other previously reported results. The setting which serves for the derivation of the bounds on the achievable rates and decoding complexity is general, and the bounds can be applied to punctured LDPC codes, non-uniformly error protected LDPC codes, and LDPC-coded modulation where all these scenarios can be treated as different forms of communication over parallel channels.

Index Terms: Block codes, complexity, low-density parity-check codes, iterative decoding, maximum-likelihood decoding, parallel channels, punctured codes, thresholds.